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An Integrated Partial Backlogging Inventory Model having Weibull Demand and Variable Deterioration rate with the Effect of Trade Credit

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Abstract - Demand considered in most of the classical inventory models is constant, while in most of the practical cases the demand changes with time. In this article, an inventory model is developed with time dependent two parameter weibull demand rate whose deterioration rate increases with time. Each cycle has shortages, which have been partially backlogged to suit present day competition in the market. Also the effect of permissible delay is also incorporated in this study. The total cost consists of ordering cost, inventory holding cost, shortage / backordering cost, lost sale cost and deterioration cost are formulated as an optimal control problem using trade credit policy. Optimal solution for the model is derived and the trade credit on the optimal replenishment policy are studied with the help of numerical examples.

Index Terms- Inventory, Shortages, partial backlogging, weibull demand, trade credit , variable deterioration, replenishment

1. Introduction

In the classical inventory economic order quantity (EOQ) model, it was tacitly assumed that the customer must pay for the items as soon as the items are received. However, in practices or when the economy turns sour, the supplier frequently offers its customers a permissible delay in payments to attract new customer. In today's business transaction, it is frequently observed that a customer is allowed some grace period before settling the accounts with the supplier or the producer. The customer does not have to pay any interest during this fixed period but if the payment gets delayed beyond the period interest will be charged by the supplier. This arrangement comes out to be very advantageous to the customer as he may delay the payment till the end of the permissible delay period. During the period, he may sell the goods, accumulate revenues on the sales and earn interest on that revenue. Thus it makes economic sense for the customer to delay the payment of the replenishment account upto the last day of the settlement period allowed by the supplier on the producer. Similarly for supplier, it helps to attract new customer as it can be considered some sort of loan. Furthermore, it helps in the bulk sale of goods and the existence of credit period serves to reduce the cost of holding stock to the user, because it reduces the amount of capital invested in stock for the duration of the credit period. So, the concept of permissible delay in payments beneficial both for supplier and customer.

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In some real life situation there is a part of demand which can not be satisfied from the current inventory, leaving the system in stock out. In these systems two situations are mainly considered, all customers wait until the arrival of the next order (Complete back order case) or all customers leave the system (lost sales case). However, in practical, some customers are able to wait for the next order to satisfy their demands during the stock out period, while others do not wish to or can not wait and they have to fill their demands from other sources. This situation is modeled by consideration of partial back- ordering in the formulation of the mathematical model. Wee (1999) developes a determinatic inventory model with quantity discount, pricing and partial backlogging when the product in stock deteriorates with time according a weibull distribution. Teng (2002) presents on EOQ model under the conditions of permissible delay. Chen et al. (2003) establish an inventory model having weibull deterioration and time varying demand. Wu et al. (2003) considered an inventory model where deteriorating rate and demand rate are follows the Chen's model (2003) where shortages are permitted. Papachristos and Skouri (2003) present a production inventory model with production rate, product demand rate and deteriorating rate, all considered as functions of the time. Their model follows shortages and the partial rate is a hyperbolic function of the time up to the order point. They propose an algorithm for finding the solution of the problem. Abad (2003) considers the problem of determining the optimal price and lot size for reseller in which unsatisfied demand is partially backordered. There are several interesting papers related to partial backlogging and trade credits viz. park (1982), Jamal et al. (1997) , Lin et al. (2000), Dye et al. (2007) and their references.

With this motivation, in this paper an attempt is made to formulate an inventory model in time dependence of demand follows a two parameters weibull type with time variable deterioration where unsatisfied

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demand is partially backlogged and delay payment is allowed. With the help of this model, supplier can easily extract its order quantity, cycle period for shortages as well as payment time to reduce the total inventory cost

2. NOTATION AND ASSUMPTIONS:

To develop the proposed model, the following notations and assumptions are used in this article.

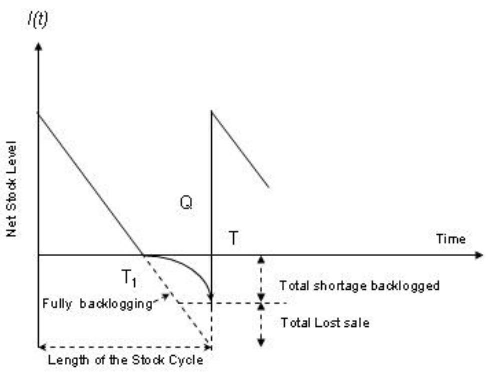
2.1 Notations

- $I(t)$: Inventory level at time t
- Q (t) : Order quantity at time t = 0
- A, h, C_1, C_2, C_3 : Ordering cost, holding cost, deteriorating cost, shortage cost for backlogged items and the unit cost of lost sales respectively. All of the cost parameters are positive constants.
- I_e : Interest which can be earned
- I_c : Interest charges which invested in inventory, $I_c \geq I_e$
- M : Permissible delay in setting the amounts, 0<M< T
- T : Length of the replenishment cycle
- T_1 : Time when inventory level comes down to zero, $0 < T_1 < T$
- TIC_1 : Total inventory cost when $M < T_1$
- TIC_2 : Total inventory cost when $T_1, < M < T$

2.2 Assumptions:

1. The inventory system involves only one item
2. Replenishment occurs instantaneously on ordering i.e. lead time is zero and it takes place at an infinite rate.
3. The demand rate of any time t is $\alpha\beta t^{\beta-1}$ two parameter weibull type, where $0 < \alpha << 1, \beta > 0$ are called scale and shape parameter respectively.
4. Q (t) = vt is the variable deterioration rate, where 0 < v <<1.
5. Shortages are allowed and unsatisfied demand is backlogged at the rate of $\dfrac{1}{1+\delta(T-t)}$

The backlogging parameter δ is a positive constant, where $T_1 \leq t < T$.



(Fig-1)

3. MODEL FORMULATION:

In this model two parameter weibull type of demand is considered with variable rate of deterioration. Depletion of the inventory occurs due to demand (supply) as well as due to inventory i.e. during the period (0,T) and shortages occurs during period (T_1, T).

The differential equation of inventory level I (t) w.r.t. time is given by

$$\frac{dI(t)}{dt} + vt\,I(t) = -\alpha\,\beta\,t^{\beta-1} \tag{1}$$

With boundary condition I (0) = Q and I (T_1) = 0

The solution of equation (1) is

$$I(t) = \frac{\alpha v}{\beta+2}\left(t^{\beta+2} - T_1^{\beta+2}\right) + \alpha\left(T_1^{\beta} - t^{\beta}\right) - \frac{\alpha v t^2 T_1^{\beta}}{2}, 0 < t < T \tag{2}$$

and $$Q = \alpha T_1^{\beta} - \frac{\alpha v \beta T_1^{\beta+2}}{\beta+2} \tag{3}$$

Similarly the differential equation for shortages is given by

$$\frac{dI(t)}{dt} = -\frac{\alpha\beta\,t^{\beta-1}}{1+\delta(T-t)} \tag{4}$$

The solution of equation (4) for shortage is

$$I(t) = \alpha t^{\beta}(\delta T - 1) - \frac{\alpha\,\beta\,\delta\,t^{\beta+1}}{\beta+1} \tag{5}$$

The deteriorating cost during one cycle is

$$DC = C_1\left[Q - \int_0^{T_1} \alpha\beta t^{\beta-1} dt\right] = \frac{-C\alpha\,v T_1^{\beta+2}}{\beta+2} \tag{6}$$

The holding cost for carrying inventory over the period (0,T) is given by

$$HC = \int_O^{T_1} h\,I(t)dt = h\left[\frac{\alpha\beta T_1^{\beta+1}}{\beta+1} - \frac{\alpha v T_1^{\beta+3}(\beta+9)}{6(\beta+3)}\right] \tag{7}$$

The total amount of shortage cost during the period (T_1, T) is given by

$$SC = -C_2\int_{T_1}^{T} I(t)dt$$

$$= C_2\left[\begin{array}{l}\dfrac{\alpha(\delta T-1)}{\beta+1}\left(T_1^{\beta+1} - T^{\beta+1}\right) + \\ \dfrac{\alpha\,\beta\delta}{(\beta+1)(\beta+2)}\left(T^{\beta+2} - T_1^{\beta+2}\right)\end{array}\right] \tag{8}$$

The amount of lost cost during the period (0,T) is given by

$$LC = C_3\int_{T_1}^{T}\left[1 - \frac{\alpha\,\beta t^{\beta-1}}{1+\delta(T-t)}\right]$$

$$= C_3\left[\frac{\delta\,\alpha\,T^{\beta+1}}{\beta+1} - \delta\alpha\,T_1^{\beta}\left(T - \frac{\beta T_1}{\beta+1}\right)\right] \tag{9}$$

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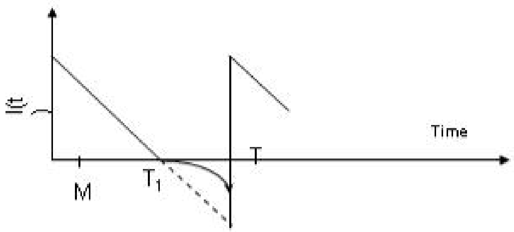
Case –I 0 < M < T1

In this situation since the length of period with positive stock is larger than the credit period, the buyer can use the sale revenue to earn interest at an annual rate I_e in $(0,T_1)$

The interest earned IE_1 , is

$$IE_1 = PI_e \int_O^M \alpha\,\beta\, t^\beta\, dt = \frac{PI_e \alpha\beta M^{\beta+1}}{(\beta+1)} \quad (10)$$



(Fig-2)

However, beyond the credit period, the unsold stock is supposed to be financed with an annual rate I_c and the interest charged IC_1 , is

$$IC_1 = PI_c \int_M^{T_1} I(t)dt$$

$$= PI_c \left\{ \left[\frac{\alpha\,\beta T_1^{\beta+1}}{\beta+1} - \frac{\alpha v\,(\beta+9)T_1^{\beta+3}}{6(\beta+3)} - \frac{\alpha v}{\beta+2}\left(\frac{M^{\beta+3}}{\beta+3} - M.T_1^{\beta+2} \right) \right] + \alpha \left(T_1^\beta M - \frac{M^{\beta+1}}{\beta+1} \right) - \frac{\alpha v\; T_1^\beta M^3}{6} \right] \quad (11)$$

Total inventory cost per unit time is given by

$$TIC_1 = \frac{1}{T}\left[A + HC + SC + LC + DC + IC_1 - IE_1\right]$$

$$= \frac{1}{T}\left[A + h\left\{ \frac{\alpha\,\beta T_1^{\beta+1}}{\beta+1} - \frac{\alpha v(\beta+9)T_1^{\beta+3}}{6(\beta+3)} \right\} + c_2\left\{ \frac{\alpha(\delta T-1)}{(\beta+1)}\left(T_1^{\beta+1} - T^{\beta+1}\right) + \frac{\delta\,\alpha\,\beta\left(T^{\beta+2} - T_1^{\beta+2}\right)}{(\beta+1)(\beta+2)} \right\} + C_3\left\{ \frac{\delta\alpha T^{\beta+1}}{\beta+1} - \delta\alpha T_1^\beta \left(T - \frac{\beta T_1}{\beta+1} \right) \right\} \right.$$

$$\left. - C_1\left\{ \frac{\alpha v T_1^{\beta+2}}{\beta+2} \right\} + PI_C\left\{ \frac{\alpha\,\beta T_1^{\beta+1}}{\beta+1} - \frac{\alpha v\,(\beta+9)T_1^{\beta+3}}{6(\beta+3)} - \frac{\alpha v}{\beta+2}\left(\frac{M^{\beta+3}}{\beta+3} - MT_1^{\beta+2} \right) - \alpha\left(T_1^\beta M - \frac{M^{\beta+1}}{\beta+1} \right) + \frac{\alpha v\, M^3 T_1^\beta}{6} \right\} - \frac{PI_e\,\alpha\beta M^{\beta+1}}{\beta+1} \right] \quad (12)$$

The solutions for the optimal values of T_1 and T can be found by solving the following equations simultaneously.

$$\frac{\partial T\, IC_1}{\partial T} = 0 \; and \; \frac{\partial TIC_1}{\partial T_1} = 0 \quad (13)$$

Provided they satisfy the conditions

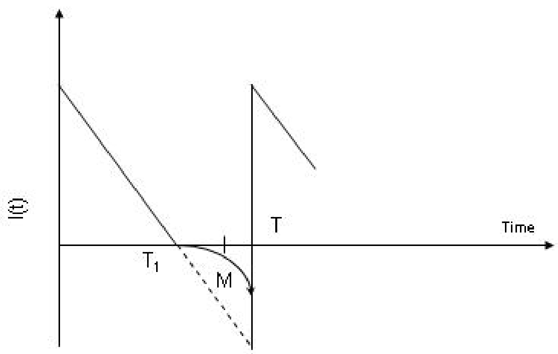
$$-\frac{\partial^2 TIC_1}{\partial T^2} > 0 \;\; and \;\; \frac{\partial^2 TIC_1}{\partial {T_1}^2} > 0 \quad (14)$$

Case – II: T > M >T1

In this situation interest charged IC_2=0 and the interest earned per time unit is

$$IE_2 = PI_e\left[\int_0^T \alpha\,\beta\, t^\beta\, dt + \alpha\beta T^\beta (M-T) \right]$$

$$= PI_e\left[\alpha\,\beta M T^\beta - \frac{\alpha\beta^2 T^{\beta+1}}{\beta+1} \right] \quad (15)$$



(Fig-3)

Then the total inventory cost per unit time is given by

$$TIC_2 = \frac{1}{T}\left[A + HC + SC + LC + DC + IC_2 - IE_2\right]$$

$$= \frac{I}{T}\left[A + h\left\{ \frac{\alpha\,\beta T_1^{\beta+1}}{\beta+1} - \frac{\alpha\, v\,(\beta+9)T_1^{\beta+3}}{6(\beta+3)} \right\} + C_2\left\{ \frac{\alpha(\delta T-1)}{\beta+1}\left(T_1^{\beta+1} - T^{\beta+1}\right) + \frac{\delta\,\alpha\,\beta\left(T^{\beta+2} - T_1^{\beta+2}\right)}{(\beta+1)(\beta+2)} \right\} + C_3\left\{ \frac{\delta\alpha T^{\beta+1}}{\beta+1} - \delta\alpha T_1^\beta\left(T - \frac{\beta T_1}{\beta+1} \right) - C_1\left\{ \frac{\alpha v T_1^{\beta+2}}{\beta+2} \right\} \right\} - PI_e\left\{ \alpha\,\beta M T^\beta - \frac{\alpha\beta^2 T^{\beta+1}}{\beta+1} \right\} \right] \quad (16)$$

The solutions for the optimal values of T_1 and T can be found by solving the following equations simultaneously.

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$$\frac{\partial^2 TIC_2}{\delta T^2}=0 \quad and \quad \frac{\partial^2 TIC_2}{\partial T_1^2}=0 \qquad (17)$$

Provided they satisfy the conditions

$$\frac{\partial^2 TIC_2}{\partial T^2}>0 \quad and \quad \frac{\partial^2 TIC_2}{\partial T_1^2}>0 \quad (18)$$

Numerical Examples:

Case I : A=50, h=2, c_1=2, C_2=0.8, C_3=2.0, α =0.0002, v=0.001, δ =0.5, p=0.5, I_c= 0.2, I_e=0.1, m =0.05

β	T_1	T	Q	TIC_1
0.8	1.90535191	106.840667	0.000335	0.718680
1.0	2.2908332	77.122890	0.000459	1.130076
1.5	2.917128	40.368203	0.001	2.662553
2.0	3.01722969	24.922125	0.001829	4.927522
2.5	2.77056753	17.154589	0.002566	7.866730
3.0	2.4129991	12.736923	0.00282	11.374538
3.5	2.07280312	9.991002	0.002571	15.332234
4.0	1.78842979	8.166399	0.00205	19.628761
4.5	1.56061254	6.889604	0.001485	24.170034
5.0	1.379736015	5.958621	0.001001	28.880925

Case II :

A=50, h-2, c1=2, C2=0.8, C3=2.0, α =0.0002, v=0.001, δ =0.5, p=0.5, Ie=0.1, m =0.5

β	T1	T	Q	TIC2
0.8	0.4998753	1.257346	0.000115	39.766547
1.0	0.41972384	1.066637	0.000084	46.876526
1.5	0.398675	1.051521	0.00005	46.962943
2.0	0.37985845	1.047747	0.000029	47.721766
2.5	0.359786885	1.007243	0.000016	49.640806
3.0	0.34985674	0.990081	0.000009	50.501263
3.5	0.29985764	0.85216	0.000003	58.674548
4.0	0.2789685	0.797059	0.000001	62.730769
4.5	0.2756759	0.791841	0.000001	63.144167
5.0	0.2679865	0.772816	00000	64.698610

4. Concluding Remarks:

In this paper, an EOQ inventory model is considered for determining the optimal replenishment schedule under variable deterioration with two parameter weibull type demand where shortages are partially back logged. Also the proposed model in- cooperates other realistic phenomenon and practical features such as trade credit period. The credit policy in payment has become a very powerful tool to attract new customers and a good incentive policy for the buyers. In keeping with this reality, these factors are incorporated into the present model. The model is very useful in retail business. In real market situations demand rate may be increased, decreased or constant. Keeping these consideration, we have assumed here that the time dependence of demand follows the two parameters weibull distributions where depending upon the values of the parameter β(<1,>1 and >2),the degree of non-linearity varies. (see Appendix I) Numerical examples are presented to justify the claim of each case of the model analysis by obtaining the optimal inventory length ,optimal economics order quantity, shortage time period and also calculated the total variable cost. From the numerical examples it is found that for high values of the parameter β, zero terminal inventory level provides higher unit of the total variable cost. Thus for simplicity regarding the applicability of the model it is not preferable to adopt zero-terminal inventory level policy.

The proposed model can be extended in numerous ways. For example we may consider the effect of inflation and time value of money by taking discount rate.

Appendix - I

The demand rate function D(t) =αβt β-1, α, β > 0 follows two parameters weibull distribution in time t.

Thus functional form represents increasing, decreasing and constant demand for different values of the parameter β

We have

$$\frac{dD(t)}{dt}=\alpha\beta(\beta-1)t^{\beta-2} \qquad (19)$$

and

$$\frac{d^2D(t)}{dt^2}=\alpha\beta(\beta-1)(\beta-2)t^{\beta-3} \qquad (20)$$

i) for 0<β<1, $\frac{D(t)}{dt}<0$ and $\frac{d^2D(t)}{dt}>0$ the demand rate decreased with time at an increasing rate.

ii) β=1, the demand rate becomes steady over time.

iii) 1< β ≤ 2, $\frac{d\ D(t)}{dt}>0$, $\frac{d^2D(t)}{dt^2}\le 0$, then the demand increases with time at decreasing rate.

Case (i) is applicable to the seasonal products towards the end of the seasons.

Case (ii) is applied to the consumer goods long established in the market by their brand name.

Case (iii) is appropriate for some fashionable product lunched in the market

If β >2 then $\frac{d\ D(t)}{dt}>0$ and $\frac{d^2D(t)}{dt^2}>0$, which implies that the demand will go on increasing with time at an increasing rate. Thus there will be an accelerated growth in demand, which is not very realistic.

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